

# Effect of Surface Waviness on Transition in Three-Dimensional Boundary-Layer Flow

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The effect of a surface wave on transition in three-dimensional boundary-layer flow over an infinite swept wing was studied. The mean flow was computed using interacting boundary-layer theory, and transition was predicted using linear stability theory coupled with the empirical  $e^N$  method. It was found that decreasing the wave height, sweep angle, or freestream unit Reynolds number, and increasing the freestream Mach number or suction level all stabilized the flow and moved transition onset to downstream locations.

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## 1. Introduction

Geometric irregularities ("roughness elements") of varying dimensions and shapes exist at different locations on aerodynamic surfaces. These irregularities strongly enhance the onset of laminar-turbulent transition and contribute to an increase in the skin-friction drag over the surface. In addition, the existence of roughness elements can lead to flow separation that results in reduced aerodynamic efficiency of the surface due to the accompanying increase in pressure drag. When roughness elements cannot be avoided their overall impact can be minimized by attempting to reduce the size of these elements. However, such a reduction may not be possible (Holmes et al., 1984). In that case, the ability to control the flow in the presence of roughness elements becomes an important consideration.

Surface roughness is unavoidable in the construction and assembly of aerodynamic surfaces. Roughness elements include screw-head slots, steps, and gaps at junctions and at the joints between the aircraft wing and the control surfaces. Additional examples include the roughness elements that arise from leading edge panels on wings, nacelles, and empennage surfaces and access panels, doors, and windows on the fuselage nose and engine nacelles (Holmes et al., 1984, 1986; Obara and Holmes, 1985). Other roughness elements can result from imperfection in the manufacturing process; examples are incorrectly installed flush rivets and surface waviness. Furthermore, roughness on aerodynamic surfaces can result from material degradation (corrosion), rain erosion, insect impingement, and icing. By performing flight and wind-tunnel natural laminar flow experiments at unit Reynolds number between  $0.63 \times 10^6 \text{ ft}^{-1}$  and  $3.08 \times 10^6 \text{ ft}^{-1}$ , Mach

numbers between 0.1 and 0.7, and leading-edge sweep angles from  $0^\circ$  to  $63^\circ$ , Holmes et al. (1984) found that some significant amount of surface waviness is acceptable on modern laminar-flow composite aerodynamic surfaces in favorable gradients of moderate strength.

Surface roughness elements can contribute to the onset of transition through the enhancement of receptivity to free-stream turbulence and acoustic disturbances; enhancement of secondary parametric excitations of both the subharmonic (Nayfeh et al., 1990; Masad and Nayfeh, 1992) and fundamental types; additional nonlinear interactions that can only be captured by the nonlinear parabolized stability equations (nonlinear PSE) or by direct numerical simulation (DNS) of the full Navier-Stokes (NS) equations (Bestek et al., 1989; Elli and Van Dam, 1991; Van Dam and Elli, 1992; Danabasoglu et al., 1993; Joslin and Grosch, 1995); and finally, the interaction between two or more of the above mentioned mechanisms. The mechanism that would account for the dominant surface roughness influence on transition in a given situation depends on both the type of flow as well as the location, size, and shape of the roughness element. Furthermore, as was pointed out by Spence and Randall (1954), the presence of multiple, closely spaced surface waves, increases the possibility of a resonance between the critical T-S frequency and the surface waviness frequency. Klebanoff and Tidstrom (1972) conducted an experiment to study the mechanisms by which a two-dimensional roughness element induces boundary-layer transition. They found sufficient evidence to conclude that the effect of a two-dimensional roughness element on boundary-layer transition can be regarded as a stability-governed phenomenon. An interesting experimental study on

transition enhancement mechanisms, including the secondary instability caused by distributed roughness, was conducted by Corke et al. (1986).

Localized surface roughness contributes to the generation of disturbances in boundary layers (boundary-layer receptivity) by providing appropriate conditions for the interaction of the free-stream acoustic or vortical disturbances with the unsteady motion of the boundary layer. As a result, the disturbances become internalized into the boundary layer. Saric and co-workers found that the receptivity of incompressible boundary-layer flow over a hump to free-stream acoustic waves increases as the hump height increases. As shown by Nayfeh and Ashour (1994), receptivity increases rapidly when the hump height was sufficient to cause separation. The TS and shear-layer instability waves in a separation bubble were found to coexist in flow over a roughness element. Although the shear-layer instability waves are associated with high frequencies, the TS waves are difficult to distinguish from the shear-layer instability waves. The Görtler vortices in flow over a roughness element develop in the concave surface regions and may interact with the TS waves. Although such interactions are weak (Nayfeh and Al-Maaitah, 1988; Malik, 1986) in two-dimensional flow over a smooth surface, this same result may not occur in the presence of a roughness element or in three-dimensional flow. The subharmonic secondary instability increased (Nayfeh et al., 1990; Masad and Nayfeh, 1992) dramatically in flow that separates due to a roughness element. In two-dimensional flows, such instability can set a three dimensionality in the flow field and can lead to early transition. The fundamental secondary instability and nonlinear interactions may play a

significant role in the breakdown to transition when the amplitudes of the disturbances are large enough to cause such interactions.

In studying the stability characteristics of flows over roughness elements that are likely to induce flow separation, a major difficulty is to obtain an accurate description of the associated mean flow. Conventional boundary-layer theory cannot be used because the abrupt geometry changes associated with the roughness element lead to strong viscous-inviscid coupling and an upstream influence, none of which are accounted for by the boundary-theory theory. Lessen and Gangwani (1976) and Singh and Lumley (1971) used approximate analytical-numerical methods to calculate the velocity profiles in flow over a roughness element. They found that the calculated profile has an inflection point. By performing temporal linear stability calculations on their calculated velocity profiles, Lessen and Gangwani (1976) showed that the roughness has a destabilizing effect and shifts the branch I neutral point toward lower Reynolds numbers, particularly at relatively large streamwise wave numbers. The methods of Lessen and Gangwani and Singh and Lumley fail if the surface distortion becomes sufficiently strong that the disturbance velocities become comparable to the unperturbed mean flow in the vicinity of the surface. In that case, the mean flow problem can be solved with a triple-deck formulation (Smith and Merkin, 1982; Smith et al., 1981), an interacting boundary-layer (IBL) theory (Davis, 1984; Ragab, 1979), or a Navier-Stokes (NS) solver.

For flow over smooth roughness elements with separating and reattaching boundary layers, the IBL can be used to obtain sufficiently accurate profiles in an efficient manner. However, if the edges of the roughness element are sharp or if its size is large



enough to induce massive separation and vortex shedding, then the triple-deck formulation and the IBL are both not applicable, and a NS solver must be used. To accurately predict the flow field with a NS solver in the presence of roughness elements that might induce separation, the grid must be fine enough so that important flow structures are not smeared by the truncation errors and the artificial dissipation. Even for the simple case of subsonic flow over a smooth flat plate with zero pressure gradient, caution must be exercised in using a NS solver to generate mean-flow profiles if a stability analysis is to be performed on these profiles (Garriz et al., 1994). Furthermore, if the number of flow cases that must be investigated in linear stability is very large, the NS calculations would be very expensive.

The mean-flow profiles generated by IBL and the stability characteristics compared well (Ragab et al., 1990) with those generated by a NS solver when a fine grid was used. The IBL was less computationally demanding than the NS solver by one to two orders of magnitude. Large discrepancies between the IBL computations and the NS results were found when a coarse grid was used for the NS computations. Moreover, the IBL was used to compute incompressible and compressible flows over smooth steps, wavy surfaces and humps, convex and concave corners, suction or blowing slots, heating or cooling strips, and finite-angle trailing edges. In these applications, separation bubbles and/or upstream influences exist; comparisons of the IBL results with solutions of the NS equations showed good agreement.

Previous investigations of the stability and transition to turbulence in boundary-layer flow over roughness elements have been primarily experimental. They have focused

primarily on determining the location of transition in a naturally occurring disturbance environment under different flow conditions. Neither the spectral content nor the growth properties of instability waves were examined. In the early experiments, the transition location was identified as the appearance of turbulent bursts downstream of a roughness element. Some of these natural transition experiments were flight experiments performed on swept and unswept wings; therefore, they included the effects of pressure gradients, compressibility, and occasionally surface suction, multiple roughness elements, three-dimensional roughness elements, and sharp roughness elements. In spite of these complications, these studies were able to provide some empirical criteria for the prediction of transition location in the flow over roughness elements. However, these criteria are valid only for the specific configurations and conditions relevant to the particular experiment. Moreover, because these criteria do not provide an understanding of the physical mechanisms involved, they cannot be used to develop techniques to control the transition process.

In this study, the effect on laminar-turbulent transition of a surface wave mounted on a swept wing was evaluated. The flow is compressible but subsonic. The effects of wave height, flow freestream Mach and unit Reynolds numbers, the wing's sweep angle, and surface suction on transition location are parameterized. linear stability theory coupled with the empirical  $e^N$  method was used for transition prediction. Furthermore, the theoretical predictions from this study are compared with the predictions of Carmichael's experimental criterion.

## 2. Formulation and Methods of Solution

### 2.1 Mean Flow

Consider the compressible subsonic flow around a single, smooth, two-dimensional wave on a swept wing. A two-parameter wave shape may be given by

$$y = y^* / c^* = (h^* / c^*) f(z) = h f(z), \quad (1)$$

where

$$z = (x^* - x_l^*) / \lambda^* = (x - x_l) / \lambda \quad (2)$$

and

$$f(z) = \begin{cases} \sin 2\pi z, & \text{if } 0 \leq z \leq 1 \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

Here,  $h^*$  is the dimensional amplitude (height) of the wave,  $\lambda^*$  is the dimensional length of the wave,  $x_l^*$  is the dimensional coordinate of the upstream (left) end of the wave. The variables  $h, \lambda, x$ , and  $x_l$  are made nondimensional with respect to the wing's chord length  $c^*$ . All of the theoretical results presented in this work are for the shape given by equation (1).

The wave under consideration can produce a separation bubble behind it when the height parameter  $h$  becomes sufficiently large. In such flows, both a strong viscous-inviscid interaction and an upstream influence are known to exist. The conventional boundary-layer formulation fails to predict such flows; therefore, one needs to use a triple-deck theory, an interacting boundary layer (IBL) theory, or a Navier-Stokes (NS) solver to analyze them. In this work, we use the IBL theory to predict the flow field.

In the IBL theory, the Prandtl transposition theorem is used with the Levy-Lees variables to obtain the nonsimilar boundary-layer equations and the corresponding boundary conditions. The upstream initial condition is a flow over a smooth surface. To account for the viscous-inviscid interaction, the inviscid flow over the displaced surface is calculated with the interaction law, which relates the edge velocity to the displacement thickness. Then, the thin-airfoil theory is used to supply the relation between the inviscid surface velocities with and without the boundary layer; it is also used to calculate the inviscid surface velocity in the absence of the boundary layer. The continuity equation is then combined with the interaction law to yield a single equation that can be solved simultaneously with the nonsimilar boundary-layer equations and boundary conditions.

## 2.2 Stability and Transition

In the stability analysis, small unsteady disturbances are superimposed on the computed mean flow quantities. Next, the total quantities are substituted into the NS equations, the equations for the basic state are subtracted out, the equations are linearized with respect to the disturbance quantities, and the quasi-parallel assumption is invoked. The disturbance quantities are assumed to have the normal-mode form so that a disturbance quantity  $\hat{q}$  is given by

$$\hat{q} = \xi(y)e^{i(\alpha x + \beta z - \omega t)} + \text{Complex conjugate} \quad (4)$$

The streamwise coordinate is  $x$ , the spanwise coordinate is  $z$ ,  $t$  is the time, and  $\alpha$ ,  $\beta$ , and  $\omega$  are generally complex. In the stability analysis, the reference length is  $\delta_r^* = \nu_\infty^* x^* / Q_\infty^*$ , with  $\nu_\infty^*$  being the dimensional freestream kinematic viscosity. The

reference velocity is the total free-stream dimensional velocity  $Q_\infty^*$ , the reference time is  $\delta_r^*/Q_\infty^*$ , the reference temperature is the freestream temperature  $T_\infty^*$ , the reference viscosity is the freestream dynamic viscosity  $\mu_\infty^*$ , and the pressure is made nondimensional with respect to  $\rho_\infty^* Q_\infty^{*2}$ , where  $\rho_\infty^*$  is the freestream density. The viscosity varies with temperature in accordance with Sutherland's formula; the specific heat at constant pressure  $C_p^*$  is assumed constant, and the Prandtl number  $Pr$  is assumed constant and equal to 0.72. For spatial stability of the flow over the infinite wing under consideration,  $\omega$  and  $\beta$  are real, and  $\alpha = \alpha_r + i\alpha_i$  is complex, in which the real part  $\alpha_r$  is the streamwise wave number and the negative of the imaginary part  $-\alpha_i$  is the spatial growth rate,  $\beta$  is the spanwise wavenumber and  $\omega$  is the disturbance frequency. The frequency  $\omega$  is related to the dimensional circular frequency  $\omega^*$  through  $\omega = \omega^* \delta_r^*/Q_\infty^*$ , which leads, with the definition of  $\delta_r^*$ , to

$$\omega = FR,$$

where

$$F = \omega^* \nu_\infty^* / Q_\infty^{*2} = 2\pi f^* \nu_\infty^* / Q_\infty^{*2} \quad (5)$$

$$R = Q_\infty^* \delta_r^* / \nu_\infty^* = x^{1/2} \text{Re}^{1/2} = \text{Re}_x^{1/2} \quad (6)$$

$$\text{Re}_x = Q_\infty^* x^* / \nu_\infty^* \quad (7)$$

and

$$\text{Re}_c = Q_\infty^* c^* / \nu_\infty^* \quad (8)$$

where  $x = x^* / c^*$  and  $f^*$  is the circular frequency in cps (Hz). Because  $\omega^*$  is fixed for a certain wave as it is convected downstream,  $F$  is also fixed for the same wave. The three-dimensional boundary layer under consideration supports both stationary and traveling disturbances. We found traveling disturbances were amplified more than stationary ones; therefore, only traveling disturbances were considered. The spanwise wavenumber parameter  $B$  is defined as

$$B = 1000\beta / R \quad (9)$$

where  $\beta = \beta^* \delta_r^*$ , and  $\beta^*$  is the dimensional spanwise wavenumber. Using the definition of  $\delta_r^*$  leads to

$$B = 1000\beta^* v_\infty^* / Q_\infty^* \quad (10)$$

and  $\beta^*$  and  $B$  remain fixed for the same physical wave.

The normal-mode form given by equation (4) separates the streamwise, spanwise, and temporal variations. The resulting ordinary differential equations and corresponding boundary conditions form an eigenvalue problem that can be solved numerically. The disturbances in three-dimensional flow are most amplified when they are three-dimensional (oblique).

To correlate the stability results with the transition onset location, we compute the integrated growth rate ( $N$  factor) along the chordwise direction. Transition is assumed to occur when the  $N$  factor reaches a certain value in the context of the  $e^N$  method. Although the  $N$  factor method has not been calibrated for the flow under consideration, an  $N$  value of 13 was used in this study to correlate transition onset. This value seems to give

results which are in reasonable agreement with the results of Carmichael's experimental criterion as will be discussed in section 3.6.

### **3. Results**

Compressible subsonic flow was assumed over a wave on a swept wing. The pressure coefficient distribution for the swept wing is shown in figure 1. The pressure gradient is favorable up to the minimum pressure point where the flow separates globally. In the next subsections, we study the effects of wave height, sweep angle, flow freestream Mach and unit Reynolds numbers, and surface suction. Our theoretical predictions are also compared with predictions of Carmichael's experimental criterion.

#### **3.1 Effect of Roughness Height**

The movement of the transition location as the height of the roughness element varies is an important consideration (Schlichting, 1979). Earlier papers on this problem assumed that the point of transition is located at the position of the roughness element when the roughness element is relatively large, or that the presence of the roughness element has no influence when it is relatively small. However, Fage (see Schlichting, 1979) has shown experimentally that the point of transition moves continuously upstream as the height of the roughness element is increased, until it ultimately reaches the position of the roughness element. Schlichting (1979) pointed out that in discussing the influence of roughness on transition, three questions must be answered. First, what is the maximum height of a roughness element below which the element has no influence on transition?

Second, what is the height of the roughness element that induces transition at the element? Third, how can the transition location be described for a roughness height in between these two limits? The answer to the first question has a significant practical aeronautical application; if such a critical height exists, then attempts can be made to keep the height of the unavoidable roughness elements below that critical level. For two-dimensional roughness element in two-dimensional flow, Masad and Iyer (1994) answered these questions using linear stability theory and the empirical  $e^N$  ( $N = 9$ ) transition criterion (Smith and Gamberoni, 1956; Jaffe et al., 1970). Thus, they correlated the transition location with the shortest distance, measured from the leading edge, at which the amplification factor ( $N$  factor) of the disturbance reached the value of 9. Their results show that the theoretically predicted transition location moves continuously as the hump height increases. This result is consistent with the experimental findings of Tani and Hama (1953). (See also Dryden, 1953.) However, this variation is not linear. The curve that describes the movement of the predicted transition Reynolds number becomes steeper as the hump height increases and becomes steepest when the flow separates. When the hump height exceeds a critical value, the location where  $N$  first reaches 9 moves slowly upstream toward a location only a short distance downstream of the center of the hump, which is the point of separation onset. Close to separation, when the predicted transition location has moved considerably upstream, the most amplified frequency increases sharply. In their experiments on roughness-induced transition, Klebanoff and Tidstrom (1972) also noted fluctuations at relatively high frequencies in the downstream vicinity of the roughness element.



In our study on the effect of a surface wave on transition in three-dimensional flow over an infinite swept wing, the  $e^N$  method ( $N=13$ ) was used to predict the transition location  $x/c$  as it varies with the wave height. Our results for the variation of predicted transition location  $x/c$  with wave height are shown in figure 2. The features of the variation in figure 2 are similar to those occurring in two-dimensional flow that were found by Masad and Iyer (1994). Namely, the transition moves gradually upstream at low heights, then it moves sharply upstream close to separation and finally it almost saturates at the location of the roughness element. The frequencies causing transition increase from  $F = 20 \times 10^{-6}$  at  $h = 0.000425$  to  $F = 40 \times 10^{-6}$  at  $h = 0.0006$ . The spanwise wavenumber parameters causing transition increase from  $B = 0.06$  at  $h = 0.000425$  to  $B = 0.1$  at  $h = 0.0006$ . The results in figure 2 are at a freestream Mach number of 0.1, a freestream Reynolds number of  $6.5 \times 10^6$ , a wave located between  $x/c = 0.2$  and  $x/c = 0.3$ , a sweep angle of  $40^\circ$ , and no suction. The flow separates when the height of the wave reaches 0.0005.

The existence of a roughness height beyond which transition takes place at the roughness element has been noted by many experimentalists (e.g., Dryden, 1953 and Fage and Preston, 1941). This height was correlated based on experimental data by defining a Reynolds number parameter  $Re_k$  (see Morkovin, 1993) such that

$$Re_k = k^* Q_k^* / \nu_k^* \quad (11)$$

where  $k^*$  is the dimensional height of roughness element,  $Q_k^*$  is the total velocity of the flow at height  $k^*$  in the absence of roughness, and  $\nu_k^*$  is the kinematic viscosity at height  $k^*$  in absence of roughness. Transition is assumed to occur at the roughness element

when  $Re_k$  exceeds a critical value. In aircraft icing studies (see, for example, Hansman, 1993), the critical value of  $Re_k$  is taken to be 600. This value is also used in subsonic boundary layer tripping studies when the ratio of roughness height to length is 1 (Braslow et al., 1966). Fage and Preston (1941) indicated that the value is above 400 for the case of flow over a circular wire mounted on a body of revolution. For the case of flow over a hemisphere on a flat plate, Klebanoff et al. (1992) found the value to be about 325.

Dryden (1953) analyzed previously published data on the effect of both single and distributed roughness on transition from laminar to turbulent flow. He collected the experimental data of Tani and Hama (1953), Tani et al. (1940), Stüper (1949), and Scherbarth (See Dryden, 1953) and showed that the ratio  $(Re_{x,tr})_{rough} / (Re_{x,tr})_{smooth}$  of transition Reynolds number on a rough plate  $(Re_{x,tr})_{rough}$  to transition Reynolds number on a smooth plate  $(Re_{x,tr})_{smooth}$  correlated reasonably well with the ratio  $k^* / \delta_{1k}^*$  of roughness height  $k^*$  to displacement thickness  $\delta_{1k}^*$  of the boundary layer at the location of the roughness element. The resulting correlation is qualitatively similar to that in figure 2, although the region in figure 2 at which transition takes place at the roughness element is missing in Dryden's figure. Dryden (1953) indicated in his comments on the correlation results that the "curve applies only when transition occurs downstream from the roughness element." Dryden also investigated the existence of a roughness height at which transition takes place at the roughness element. In analyzing their experimental data, Tani and Hama (1953) indicate that "departures from a single functional relation between  $(Re_{x,tr})_{rough}$  and  $k^* / \delta_{1k}^*$  occurred as the transition position approached the position of the roughness

element." The existence of two functional relations between the predicted transition Reynolds number and the hump's height is clear in figure 2.

### **3.2 Effect of Sweep Angle**

To study the effect of a wing's sweep angle on transition in flow over a wave, we considered a wave of height 0.00055 located between  $x/c = 0.2$  and  $x/c = 0.3$ . The freestream Reynolds number is  $6.5 \times 10^6$  and the freestream Mach number is 0.1. The sweep angle was varied in the range from  $\Lambda = 20^\circ$  to  $\Lambda = 65^\circ$ . Variation of predicted transition onset location with sweep angle is shown in figure 3. It is clear from figure 3 that increasing the sweep angle destabilizes the flow and moves transition to upstream locations.

### **3.3 Effect of Unit Reynolds Number**

Another parameter of importance that affects the location of transition in a flow over a roughness element is the flow unit Reynolds number. Morkovin (1969) pointed out that the effect of the unit Reynolds number on the stability characteristics of any flow is always a factor whenever the mean flow is nonsimilar.

To study the effect of unit Reynolds number on transition location, we considered a Mach 0.1 flow over a wave located between  $x/c = 0.2$  and  $x/c = 0.3$  and of height = 0.00055. The wave is on a swept wing with the sweep angle equal to  $40^\circ$ . The transition onset location was predicted as a function of Re using the  $e^N$  method with

$N = 13$ . The variation of predicted transition onset location with freestream Reynolds number is shown in figure 4. It is clear from figure 4 that the transition moves upstream as the unit Reynolds number increases. This variation agrees qualitatively with the flight data of Holmes et al. (1986). While explaining the strong beneficial effect of higher altitudes on allowable step heights and gap lengths, they noted that, "The increases in tolerances with increased altitude result directly from the decrease in unit Reynolds number. As the unit Reynolds number decreases, the length of the laminar separation regions associated with the steps decreases, reducing the growth of the inflectional instability and increasing the allowable step height." Maddalon and Braslow (1992) performed flight experiments on a Jet Star airplane to investigate the effect of 2-D forward- and backward-facing steps mounted near and parallel to the leading-edge of a 30 degree swept wing. During a part of those flights, the unit Reynolds number was varied and the corresponding movement in transition location was recorded. The recorded variation was found to be similar to the corresponding variation in figure 4. In fact, a similar effect of unit Reynolds number has also been noted in the context of the low-speed flow over a micron-sized three-dimensional roughness element on a swept wing (Radeztsky et al., 1993). Mochizuki (1961) performed experiments to examine how the flow patterns around a sphere mounted on a flat plate change with unit Reynolds number. The diameters of the spheres used by Mochizuki were 0.71, 0.55, 0.33 and 0.23 cm. The roughness element was set at various distances from the leading edge of the plate and the values of  $Re_k$  varied in a range roughly from 700 to 1000. Mochizuki (1961) indicated that "As the velocity is further increased, the wedge-shaped turbulent region appears

downstream and gradually approaches the sphere, encroaching upon the laminar part."

This observation is also in agreement with the results in figure 4.

For the wave on the swept wing in this study, increasing the unit Reynolds number enhances separation. For example, at  $Re = 5 \times 10^6$ , the flow separates at  $x/c = 0.256$  and reattaches at  $x/c = 0.273$  with a maximum flow reversal of 0.37%. Increasing  $Re$  to  $6.5 \times 10^6$ , the flow separates at  $x/c = 0.256$  and reattaches at  $x/c = 0.277$  with a maximum flow reversal of 0.86%.

### **3.4 Effect of Compressibility**

The effect of compressibility on the stability characteristics of flow over roughness elements is complicated by the fact that although an increasing Mach number stabilizes the flow in the attached regions, it increases the size of the separation bubble. An increase in the value of the free-stream Mach number  $M_\infty$  at subsonic and supersonic speeds causes the flow over the roughness to separate at lower heights because compressibility makes the pressure gradient more adverse and enhances separation. When the flow separates, increasing the free-stream Mach number increases the length of the separation bubble by shifting the separation location upstream and shifting the reattachment location downstream. In their experimental work, Larson and Keating (1960) noticed a large increase in the streamwise length of the separation region when the Mach number of the flow over the roughness element was increased. Note that what Larson and Keating (1960) refer to as the transition Reynolds number in the case of separation is actually the product of the flow unit Reynolds number and the streamwise length of the separation

bubble. Therefore, at the same unit Reynolds number, an increase in what they refer to as the transition Reynolds number is actually an increase in the streamwise length of the separation bubble. For the flow over a wave on a swept wing under consideration, increasing Mach number is found to increase the size of the separation bubble. For example, at a sweep angle of  $40^\circ$ , a freestream Reynolds number of  $6.5 \times 10^6$  and a 0.00055 height wave located between  $x/c = 0.2$  and  $x/c = 0.3$ , the flow at  $M_\infty = 0.1$  separates at  $x/c = 0.256$  and reattaches at  $x/c = 0.277$  with a maximum flow reversal of 0.86%. Whereas at  $M_\infty = 0.4$ , the flow separates at  $x/c = 0.253$ , reattaches at  $x/c = 0.280$ , and has a maximum flow reversal of 1.44%.

The widening of the separation region because of the increase in  $M_\infty$  partially offsets the stabilizing effect of compressibility. Overall, the stabilizing effect of compressibility in the attached regions overcomes the destabilization caused by the increase in the size of the separation bubble (figure 5). The downstream movement of the transition location of a flow over a step as the Mach number increases was noticed and reported by Chapman et al. (1958). Van Driest and Boison (1957) experimentally studied the effect of 2-D surface roughness (circular wire) mounted on a cone on transition at supersonic Mach numbers. They indicated a "spectacular role of Mach number in damping the effect of roughness on transition." They also indicated that "with increasing Mach numbers, increasingly large ratios of roughness height to boundary-layer displacement thickness were necessary to promote transition." Furthermore, the stability of a laminar shear layer (that develops in the case of separation) was found by Lin (1955) and Gropengiesser (see Morkovin, 1987) to increase markedly as the Mach number

increases. At supersonic speeds in wind-tunnel operation, larger wire diameters are required to trip the boundary layer (make it turbulent) as the Mach number increases. (See, for example, Coles, 1954.) Brinich (1954) performed measurements on the effect of cylindrical roughness elements of circular cross-section on transition at a Mach number of 3.1. The measurements showed that at high Mach numbers, the boundary layer can "tolerate" a considerably larger roughness element than in incompressible flows. Experiments performed by Korkegi (1956) at the even higher Mach number of 5.8 showed that at such large Mach numbers a tripping wire produces no turbulence at all.

For air boundary-layer flow, as the Mach number increases, the adiabatic wall temperature also increases. At hypersonic Mach number, the adiabatic wall temperature reaches very high value. Existing metallic and composite materials cannot withstand some of these high temperatures. Movkovin (1987) indicated that under the extreme heat generated in hypersonic flight surface roughness could result from local buckling, swellings, gaps, or erosion of surface. At such high temperatures, the materials must be thermally protected, which can be achieved by cooling the surface.

Von Doenhoff and Braslow (1961) found experimentally that the three-dimensional roughness height required to influence transition at supersonic speeds up to a Mach number of 2 is greater than at subsonic speeds. They attributed the difference to the boundary-layer thickening effect of increasing Mach number. Sparse data at a Mach number of 3.5 (Carros, 1956) confirm the same trend. The experimental data of Von Doenhoff and Braslow (1961) at supersonic speeds were for three-dimensional roughness on plates and cones.

### 3.5 Effect of Continuous Uniform Suction

Although continuous suction thins the boundary layer (which makes the boundary layer more sensitive to roughness), continuous suction also reduces the size of the separation bubble. In fact, suction can be used in applications to remove the decelerated fluid from the boundary layer before it causes separation. This technique makes the boundary layer capable of overcoming a stronger adverse pressure gradient. The reduction in the size of the separation bubble by suction was observed and reported in the experimental work of Hahn and Pfenninger (1973) for the case of flow over a backward-facing step. For the flow over a wave on a swept wing under consideration, applying suction is found to decrease the size of the separation bubble. At a sweep angle of  $40^\circ$ , a freestream Reynolds number of  $6.5 \times 10^6$ , a freestream Mach number of 0.1, and a wave located between  $x/c = 0.2$  and  $x/c = 0.3$  and of height 0.00055, the flow with no suction separates at  $x/c = 0.256$  and reattaches at  $x/c = 0.277$  with a maximum flow reversal of 0.86%. With continuous uniform suction of  $v_w = -5 \times 10^{-5}$ , the flow separates at  $x/c = 0.260$  and reattaches at 0.270 with a maximum flow reversal of 0.20%.

Continuous uniform suction might affect the flow in the separation region differently than the flow in the attached regions. This possibility might be attributed to the coexistence of both viscous and shear-layer instability mechanisms in the separation region, whereas in the attached regions only the viscous instability mechanism exists. Although continuous suction might increase the growth rate of disturbances within the reduced separation bubble, the overall effect of continuous suction on transition in flow



over a wave on a wing is stabilizing (figure 6). Carmichael (1957) and Carmichael and Pfenninger (1959) performed flight experiments on the wing of an airplane in the presence of single and multiple roughness elements and suction. Their results show that the allowable sizes of the roughness elements increase when embedded in the suction region.

### **3.6 Comparison of Transition Prediction Results With Experimental Criterion**

An experimental correlation for transition in flow over single or multiple waves on a wing is given by Carmichael (1957). Carmichael's criterion applies for single and multiple bulges or sinusoidal waves above the nominal surface of a swept or unswept wing. Carmichael's criterion partially accounts for the effects of compressibility, suction, pressure gradient, wing sweep, and multiple waves, which makes a quantitative comparison of theoretical results with this criterion a difficult task. However, a quantitative comparison of the results of Masad and Iyer (1994) from the  $N$ -factor criterion with the predictions of Carmichael's criterion for unswept wings showed that those transition locations predicted by the  $N$ -factor method are upstream of those predicted by Carmichael's criterion. This result is expected because Carmichael's data base involved varying effects of compressibility, suction, and favorable pressure gradient on the unswept wing; these effects which were not included in the calculations tend to move the transition location downstream.

Carmichael's criterion in its general form is valid for 2-D or 3-D flow over 2-D or 3-D roughness in the form of single and multiple chordwise and spanwise waves. For wing flow over a 2-D chordwise surface wave, Carmichael's criterion is given by

$$\hat{h}^* / \lambda = 59000^5 c^5 \cos \Lambda / \lambda^5 \text{Re}_c^{0.75} \quad (12)$$

where  $\hat{h}^*$  is the dimensional double-amplitude wave height,  $\lambda^*$  is the dimensional wavelength,  $c^*$  is the dimensional chord length,  $\Lambda$  is the wing sweep angle, and  $\text{Re}_c$  is the freestream Reynolds number based on the chord length, therefore,

$$\text{Re}_c = Q_\infty^* c^* / \nu_\infty^* \quad (8)$$

The value of  $\hat{h}^* / \lambda^*$  given by equation (12) determines the critical value of the surface wave. Carmichael defined this critical value of the wave as the minimum  $\hat{h}^* / \lambda^*$  which prevents the attainment of laminar flow to the trailing edge. It follows from this definition that the value of  $\hat{h}^* / \lambda^*$  given by equation (12) is the value that just causes transition at the trailing edge. Therefore, we have

$$x_t^* = c^* \quad (13)$$

and

$$\text{Re}_{x,tr} = \text{Re}_c \quad (14)$$

where

$$\text{Re}_{x,tr} = Q_\infty^* x_t^* / \nu_\infty^* \quad (15)$$

We also have

$$\hat{h}^* = 2h^* \quad (16)$$

Using relations (13)–(16), equation (12) can be rewritten as

$$\text{Re}_{x,tr} = 601 \Gamma, \quad \text{single wave} \quad (17)$$

where

$$\Gamma = \lambda^{2/3} \cos^{4/3} \Lambda / h^{4/3} \quad (18)$$

For multiple waves, Carmichael proposed multiplying the right-hand side of equation (12) by 1/3. This results in

$$\text{Re}_{x,c} = 139 \lambda^{2/3} \cos^{4/3} \Lambda / h^{4/3}, \quad \text{multiple waves} \quad (19)$$

Holmes et al. (1986) indicated that this multiple-waves correlation "was developed using closely spaced waves and does not address any effects due to widely spaced waves." Holmes et al. (1986) added that "closely spaced waves may have T-S resonance effects which might be less likely to occur for widely spaced waves. Furthermore, the wind-tunnel and flight experimental results used to develop the factor of 1/3 actually varied over a range from 1/3 to 3/4, with the flight values being typically greater than the wind-tunnel values. Thus, some uncertainty exists concerning a realistic method for figuring the effect of multiple waves on the allowable  $(\hat{h}^* / \lambda^*)$ ." The experimental data points which are the basis for Carmichael's criterion included effects of compressibility, suction, and pressure gradient. Since these factors have a stabilizing effect, it is expected that the transition Reynolds number predictions of Carmichael's criterion will approximately constitute an upper bound for the theoretical predictions. Holmes et al. (1986) indicated that in several flight experiments (Holmes et al., 1984) the measured aircraft surface waviness was found to be better than required by Carmichael's criterion using the single-wave assumption. Holmes et al. (1986) added that "since the allowable waviness values were calculated for the low altitudes and high speeds of the flight experiments, the allowable waviness at lower Reynolds numbers for typical cruise conditions for all of the airplanes will be even larger." Our work indicates that although low altitudes result in higher unit Reynolds numbers which has a destabilizing effect, a high subsonic speed has a stabilizing effect.

Therefore, the combined effect of increasing the altitude and reducing the speed depends on the contribution of each of the two counter effects. However, Holmes et al. (1986) concluded from the above inaccurate argument and the indicated flight experiments that a conservative value for allowable waviness on unswept (sweep angle  $< 15^\circ$ ) natural laminar flow (NLF) wings can be determined using Carmichael's criterion for a single wave. In the same work where the above argument about the compressibility effect was made by Holmes et al. (1986) they indicate that "compressibility influences allowable waviness in two ways. First, compressibility favorably increases the damping of growth rates for T-S waves. The second unfavorable effect results from the increased pressure peak amplitude over a wave due to compressibility. It is not clear which effect dominates." We now know from section 3.4 that the net effect of compressibility is stabilizing. However, from their above argument, it is not clear how Holmes et al. (1986) concluded that compressibility is destabilizing.

Carmichael's criterion also applies for 3-D roughness in the form of a spanwise wave which has its peak and valley aligned in the chordwise direction in 2-D flow. For this configuration the recommendation of Anon (1967) is to multiply the right-hand side of equation (12) by 2. Using a derivation similar to the one we performed earlier, it can be shown that

$$\text{Re}_{x,tr} = 1514 \lambda^{2/3} \cos^{4/3} \Lambda / h^{4/3}, \text{ spanwise single wave} \quad (20)$$

and for multiple waves,

$$\text{Re}_{x,tr} = 350 \lambda^{2/3} \cos^{4/3} \Lambda / h^{4/3}, \text{ multiple spanwise waves} \quad (21)$$

The X-21 experiments (Anon, 1967) determined a gap critical Reynolds number  $Re_{h,crit} = U_{\infty}^* h^* / \nu^*$ , where  $h^*$  is the critical width of the gap and the flow is along the gap. The critical width of the gap was defined as the width at which the first turbulent bursts occurred far downstream from surface imperfection. For this configuration,  $Re_{h,crit}$  was determined to be 2143, which is 1/7 of the value for flow across a gap. Braslow et al. (1990) pointed out that flow along a gap should definitely be avoided. This criterion does not account for the effect of gap depth. Furthermore, if the gap is of finite length in the chordwise direction, then the criterion does not account for the effect of location of the gap. Finite-length gaps could result from metal scratches which occurred in the flight tests on a modified Jet Star airplane within the Leading Edge Flight Tests (LEFT) Program (Maddalon, personal communication, 1994).

#### 4. Conclusions

The effect on laminar-turbulent transition of a surface wave mounted on a swept wing was studied. The effects of wave height, freestream Mach and unit Reynolds numbers, wing's sweep angle, and surface suction on transition were evaluated. The mean flow was computed using interacting boundary layer (IBL) theory, and transition was predicted using linear stability theory coupled with the  $e^N$  method with  $N = 13$ . Based on this study, the following conclusions were reached:

1. The variation of transition onset location with wave height is characterized by gradual upstream movement of transition location at low wave heights, followed by sharp upstream movement close to separation, and finally by a near saturation of transition location at the wave's location.
2. Increasing the sweep angle is found to destabilize the flow.
3. Increasing the unit Reynolds number is found to enhance separation and to move transition to upstream locations.
4. Increasing the freestream Mach number is found to enhance separation, but to move transition to downstream locations.
5. Suction is found to reduce the size of the separation bubble and to move transition to downstream locations.
6. The results of Carmichael's experimental criterion are found to almost form an upper bound on the theoretical prediction results. This result is reasonable because the experimental data base points for Carmichael's

criterion include effect of suction, compressibility, and favorable pressure gradient.

7. Results on the effects on transition of wave height, flow freestream Mach and unit Reynolds numbers, and surface suction are found to be consistent and in agreement with wind-tunnel and flight observations.

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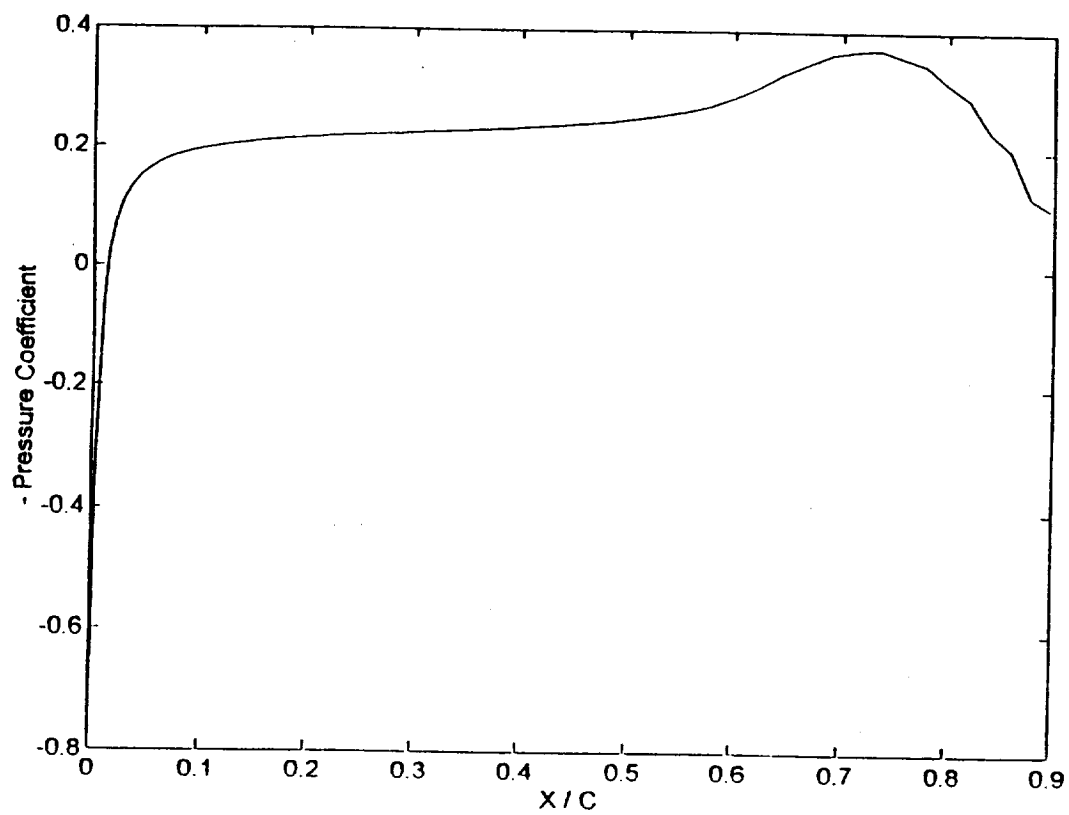


Figure 1. Chordwise distribution of pressure coefficient in flow over a smooth wing.  
 $\Lambda = 40^\circ$  and  $M_\infty = 0.1$ .

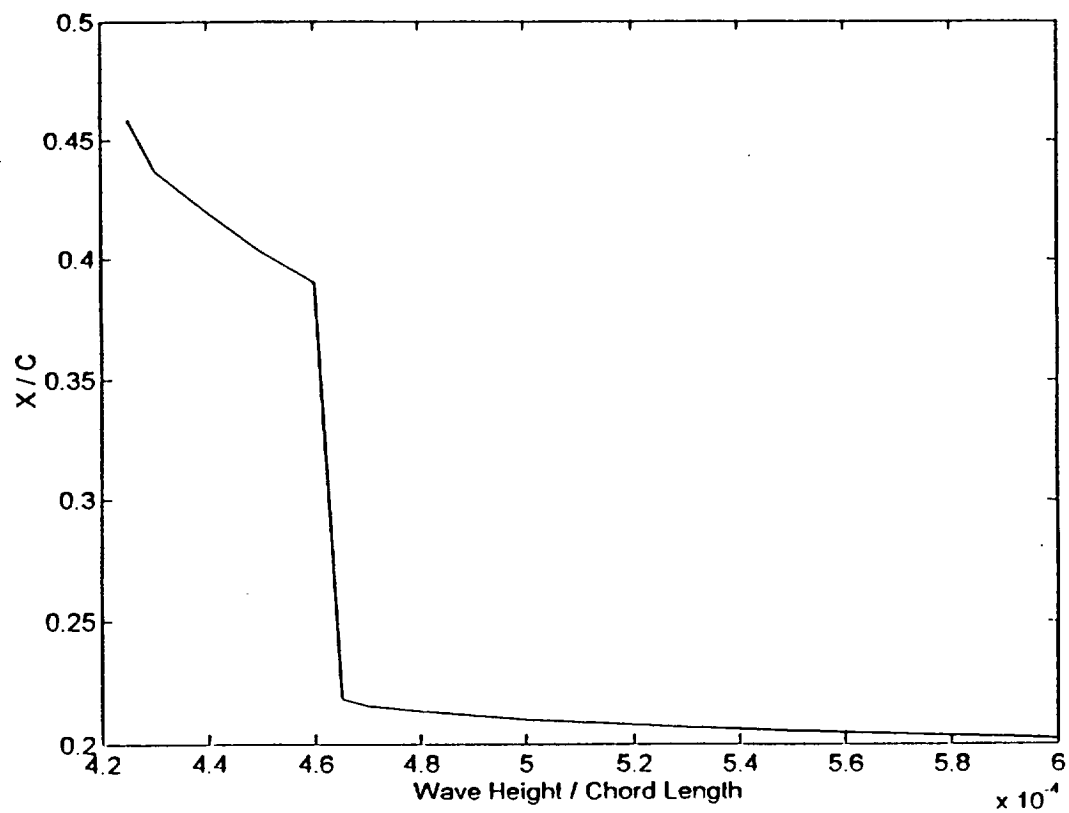


Figure 2. Variation of predicted transition onset location with wave height.  $\Lambda = 40^\circ$ ,  $Re_c = 6.5 \times 10^6$ ,  $M_\infty = 0.1$ , and wave located between  $x/c = 0.2$  and  $x/c = 0.3$ .

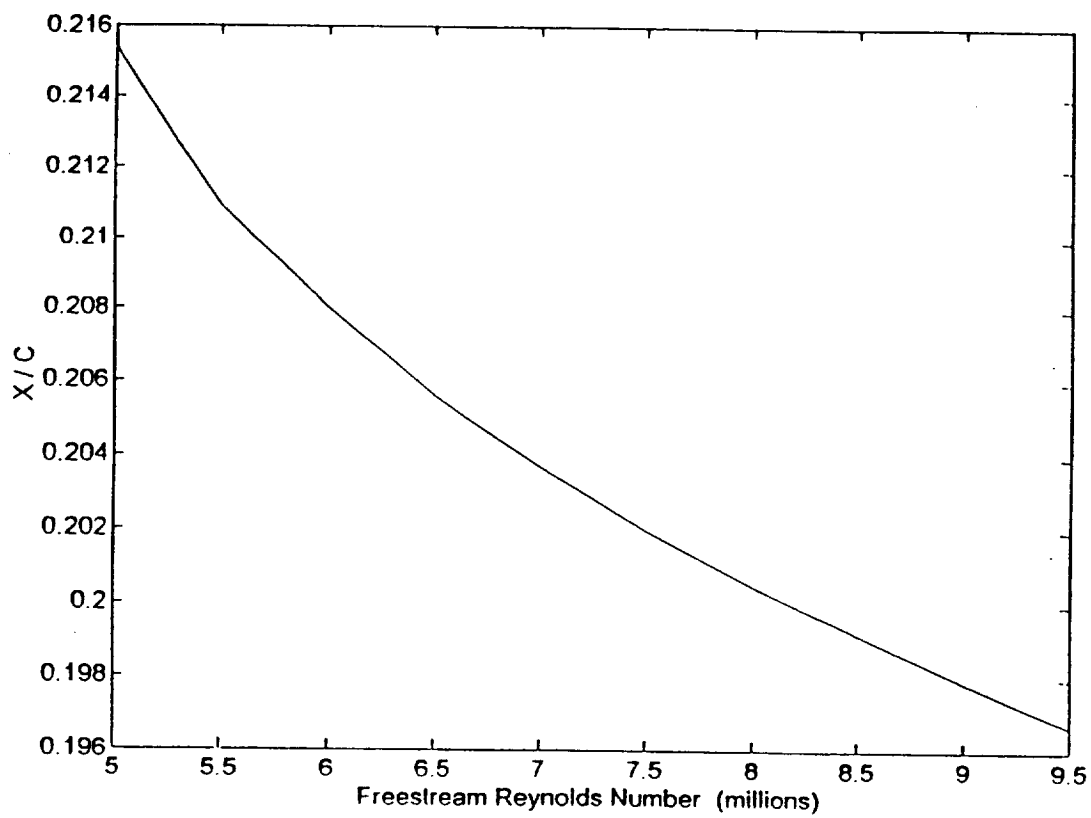


Figure 3. Variation of predicted transition onset location with sweep angle.  $Re_c = 6.5 \times 10^5$ ,  $M_\infty = 0.1$ ,  $h = 0.00055$ , and wave is located between  $x/c = 0.2$  and  $x/c = 0.3$ .

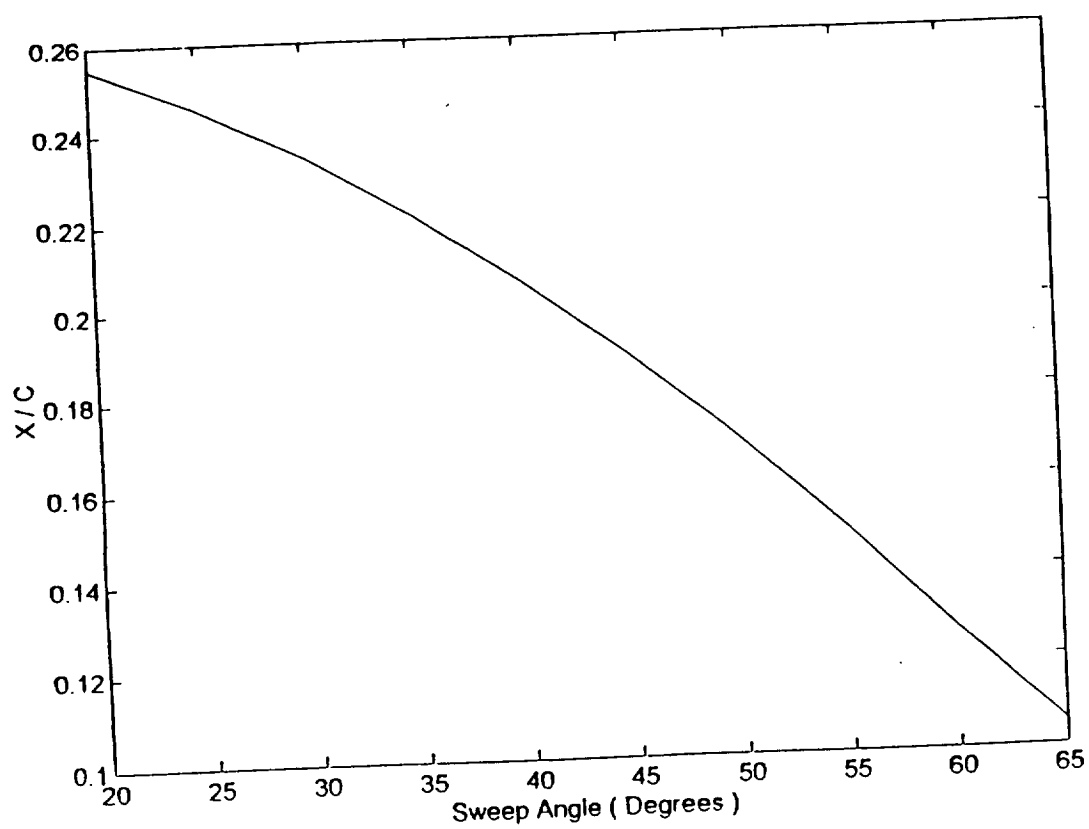


Figure 4. Variation of predicted transition onset location with freestream Reynolds number.  $\Lambda = 40^\circ$ ,  $M_\infty = 0.1$ ,  $h = 0.00055$ , and wave is located between  $x/c = 0.2$  and  $x/c = 0.3$ .

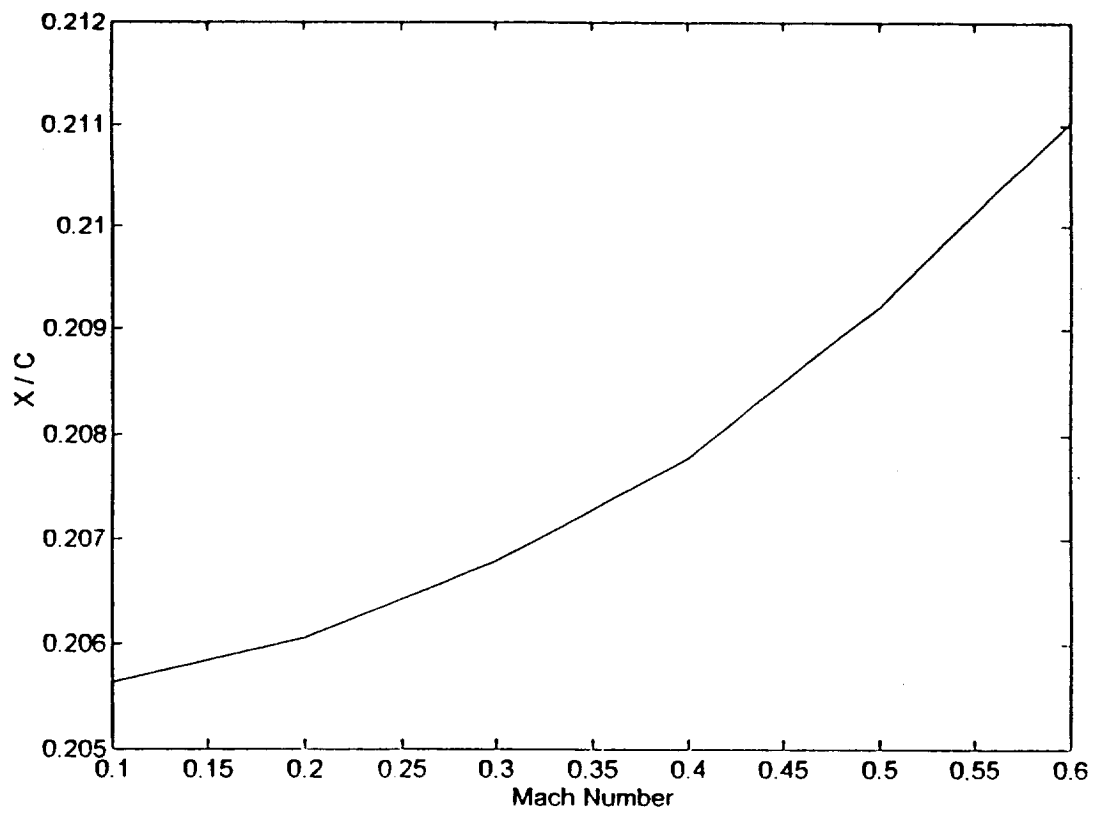


Figure 5. Variation of predicted transition onset location with freestream Mach number.  $\Lambda = 40^\circ$ ,  $Re_c = 6.5 \times 10^6$ ,  $h = 0.00055$ , and wave is located between  $x/c = 0.2$  and  $x/c = 0.3$ .



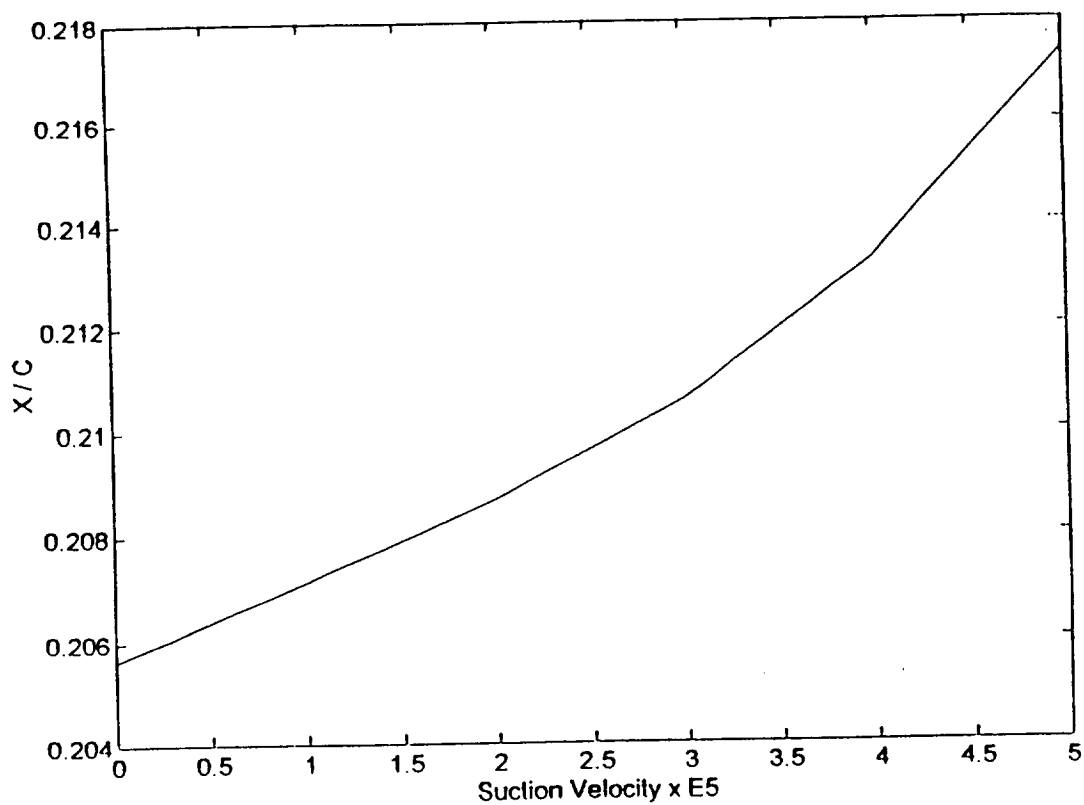


Figure 6. Variation of predicted transition onset location with suction velocity  $v_w = v_w^* / Q_\infty^*$ .  $\Lambda = 40^\circ$ ,  $Re_c = 6.5 \times 10^6$ ,  $M_\infty = 0.1$ ,  $h = 0.00055$ , and wave is located between  $x/c = 0.2$  and  $x/c = 0.3$ .

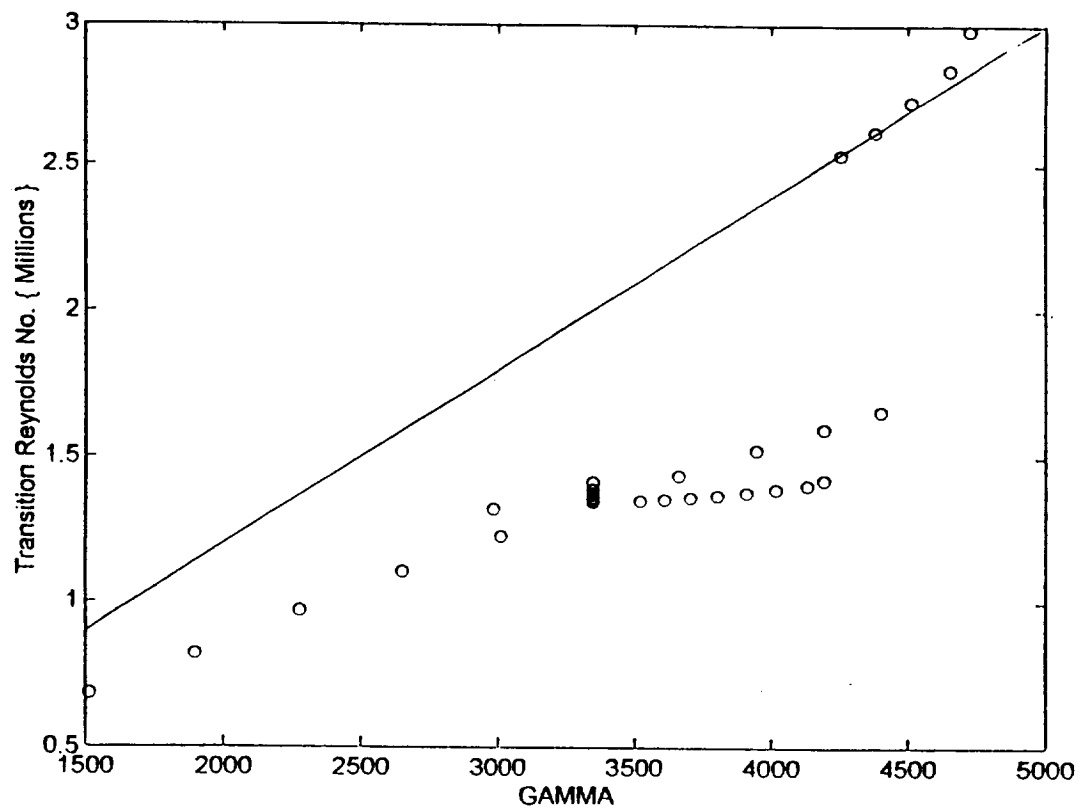


Figure 7. Variation of transition Reynolds number with the parameter  $\Gamma$  (equation 18).  
 ( ---- ) Carmichael's criterion and (  $\circ$  )  $e^N$  method with  $N = 13$ .



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